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MATHEMATICAL MODEL FOR DETERMINING THE OPTIMAL DOSES OF MINERAL FERTILIZERS FOR WHEAT CROPS

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The intensiveness of agricultural systems is given by the sum of the production factors provided per time unit and area. In this sense, fertilizers represent one of the factors with high variability in the intensification process. In agricultural practice, in order to maintain high yields, the fertilizers that are commonly applied contain either one or more macro elements (NPK). In the unifactorial model proposed in the paper, which starts from a Mitscherlich-type model, the constants were determined and then the concordance was checked between the experimental data and the theoretical data on a wheat crop. Afterwards, the optimum of the benefit was studied. Subsequently, using a function which generalizes empirically to two variables, good concordance was also found between the theoretical and experimental data. The optimum benefit was determined by employing the least squares method in calculating the constants. By hypothesis, the bifactorial model takes into consideration all three nutritive elements: nitrogen, phosphorus and potassium, but considers them as two independent variables, namely x for nitrogen and y for the phosphorus-potassium complex in equal proportions.
Keywords: Economic indicators, Mitscherlich, optimal benefit, prediction model, macro elements

INTRODUCTION

Plant nutrition is a complex physiological process. Fertilizers play a significant role in obtaining agricultural yield, with a variable contribution between 40-60% depending on the crop, the climatic conditions and agricultural technology (FAO report, 2000; Stewart *et al*., 2005).
Plant feeding takes place in optimal conditions when macronutrients and micronutrients are adequately provided in balanced quantities in relation to the needs of plants (Mengel and Kirkby, 2001). The degree of participation in the final yield is different for each nutrient. Nitrogen nutrition played an important role on growth, yield components and grain quality of cereal crops (Marschner, 1995; Epstein and Bloom, 2005; Maqsood *et al*., 2014).
Smil (2002) estimates that fertilizer N has contributed an estimated 40 percent to the increases in per-capita food production in the past 50 years, although there are local and regional differences and varying efficiencies. Due to the high importance of nitrogen in agricultural production, soil dynamics, fertilization efficiency and the impact it has on the environment, it is one of the most widely studied nutrients for different crops (Makowski and Wallah, 2001; Marchetti and Castelli, 2011; Basso *et al*., 2013). Because of the importance of wheat crop, much research aimed at the relationship between wheat and fertilizer under different conditions (Shah *et al*., 1996; Tahir *et al*., 2003; Wajid *et al*., 2007; Ballesta and Lloveras, 2010; Villar and Guillaumes, 2010; Qamar *et al*., 2012; Hammad *et al*., 2013).
Modeling of agricultural systems has been used for over several decades, over this period of time, a series of models and approaches have been created. In the past, the modeling process used to be quite difficult; it involved a large amount of work, while only dealing with simple processes and phenomena (Ahuja *et al*., 2002).
With regard to soil-plant interactions, the first theories and models were formulated and developed for the analysis of evapo-perspiration (Penman, 1848; Monteith, 1965), photosynthesis (Saeki, 1960) and root growing (Forth, 1962; Shaffer *et al*., 1969). In the early 1970s, a few models were designed for the complex approach of agricultural systems, by including multiple components of the researched subsystems (Dutt *et al*., 1972). Nevertheless, modeling of systems and agricultural technology was not developed and used on a large scale before 1980s.
Information technology (IT) contributed to the development of modeling for the analysis of crops and farming systems as a whole, Windows or OS platforms offered interfaces that facilitate data management and development of simulation models for different specialized applications: APES (Donatelli *et al*., 2010), CROPSYST (Stockle *et al*., 2003), DAISY (Abrahamsen and Hansen 2000; Hansen *et al*., 1990; Hansen 2000), DSSAT (Ritchie and Otter 1985;

Hoogenboom *et al*., 2003; Jones *et al*., 2003), FASSET (Olensen *et al*., 2002a, 2002b; Berntsen *et al*., 2003), HERMES (Kersebaum 1995; Kersebaum and Beblik 2001; Kersebaum 2007), SALUS (Basso *et al*.. 2011), STICS (Brisson *et al*., 1998; Brisson *et al*., 2003; Brisson *et al*., 2009), WOFOST (van Diepen *et al*., 1989; Supit *et al*., 1994; Boogaard *et al*., 1998).

The soil-plant relationship, with direct reference to nutrition and nutrient consumption, was presented by Mitscherlich early in the twentieth century through mathematical relations that have served as the foundation of many current models.

Designing computer models of the soil-plant and fertilizer-yield relationship, respectively, enjoys considerable popularity, as this type of research is considered an important tool for the analysis and prognosis of fertilization efficiency, plant nutrition and agricultural system productivity (Cooke, 1998).

A series of mathematical relations and models have been developed in order to assess the relation between fertilization and yield, both from a technical point of view and from an economic one. These capture and express the level of interdependence between the two variables, with varying degrees of accuracy (Mitscherlich, 1909; Borlan *et al*., 1982; Makowski *et al*., 1999; Budoi, 2004; Anghel and Boldea, 2007; Finger and Hediger, 2007; Aizpurua *et al*., 2010; Boldea and Sala, 2010; Sala and Boldea, 2011).

Over time, the models used have been unifactorial, namely second order polynomial models. New tendencies in mathematical modeling start from differential equations, the solutions of which are continuous functions. These are valuable help for making predictions in relation to any level of the parameter taken into consideration. Also, passing from the unifactorial model to the bifactorial model and then to the multifactorial model is a common trend in agricultural research. It is the trend, that's the valuable research of Harmsen (2000a) and Harmsen *et al*. (2001) were framed. Nijlandl *et al*. (2008) succeed in integrating such models, the Mitscherlich model among others, in a modern, dynamic system.

Recent research focuses on the prediction accuracy of models that are widely used in research and agricultural practice in order to calibrate and validate the procedures and methods for increasing the precision of prediction (Guillaume *et al*., 2011; Palosuo *et al*., 2011; Wang *et al*., 2013). Optimizing fertilization also requires constant research due to the need to adapt plant growing technology to climatic and socio-economic conditions always changing.

The aim of this research is to study the relationship between fertilizer and yield for the winter wheat crop, in an attempt to establish the optimal doses in the cases where the influence is unifactorial (N fertilizer) and also where there is a multifactorial influence (NPK fertilizers). The guidelines for developing the mathematical model used in this research to assess the fertilizer-yield relationship were the practical measures proposed by Cooke (1998) for general formulation of mathematical models.

MATERIALS AND METHODS

In the evaluation of the relationship between fertilizers and crop, for determining the optimal doses through mathematical modeling, the results of an experiment on winter wheat fertilized with different doses of mineral fertilizers were used. The impact of nitrogen was estimated as the single nutrition factor, in addition jointly with three different levels of phosphorus and potassium. The optimum dose of fertilizer application was estimated from a technical and economic point of view, in terms of the maximum advantage.

The reference area for the study is in the West Plain of Romania, more specifically in Timiş County. This county has the largest agricultural surface area in the country, 747,000 ha and 531,373 ha arable land (Fig. 1).

This condition has determined systems based on simple wheat-maize rotations or even on monoculture. Under these circumstances, nutritional support for crops is almost entirely based on mineral fertilization; simple nitrogen fertilizers are repeatedly used, because they are cheap. These aspects are all revealed by the statistics on fertilizer use (Romanian Statistical Yearbook, 2008). In this context, the present research was focused on determining the optimal doses for wheat through mathematical modeling.

From a geological point of view, the area of the experiment is part of the vast Pannonian Depression, the eastern extremity, which was formed by gradual clogging of the lake in Pleistocene-Quaternary. The natural setting of the experiment is specific for Banat Plain, with slightly gleized cambic chernozem with neutral reaction (*p*H = 6.7-6.8), good humus supply (H = 3.2%), nitrogen index IN = 2.78, high degree of base saturation (over 87%) supply of phosphorus (P = 10.23 ppm) and medium supply of potassium (K = 132 ppm).

***Fertilizer application and experimental variant*:** The experimental treatments consisted of five nitrogen fertilizer rates (0, 50, 100, 150 and 200 kg N ha^{-1}) and four levels of complex (P:K) fertilizer (P_0K_0, $P_{50}K_{50}$, $P_{100}K_{100}$, $P_{150}K_{150}$ kg P:K ha^{-1}).

The wheat cultivated was *Alex* cultivar, which has high productivity, biological yield potential of 7500-8000 kg ha^{-1}; this cultivar is recommended for the West part of the country and Timiş County. The biological potential of the Alex cultivar was not reached under the conditions of the experiment (the variation interval of fertilizer rates), but conditions for crop variation were created for determining the experimental coefficients useful for the design of the mathematical model.

***Climatic conditions*:** According to Timisoara Meteorological Station, the climate conditions are

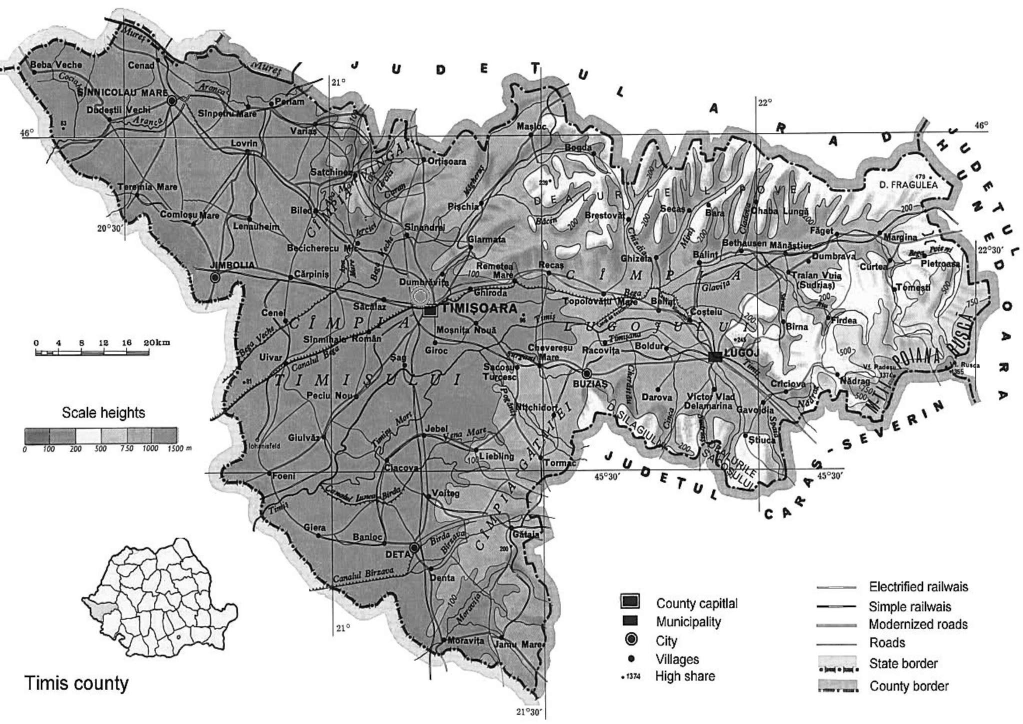

Figure 1. Timiș County area, Romania – the reference area for study and research

characterized by multiannual average values of 625 mm rain fall and temperatures of 10.9°C. Under the current circumstances of climate changing, the distribution of rainfall is not uniform, with deficits especially in June - July - August, but sometimes also in September and October. Temperatures are higher than the multiannual mean, reaching 11.3°C.

Mathematical model and function: The mathematical instruments were designed starting from the Mitscherlich relation which defines the relation between nutrient and yield. For the situations when only one nutrient is provided through fertilization, the best assessment of the yield dependence on the dose of fertilizer is made with the help of the relation given by Mitscherlich.

$$f(x) = f(0) + a(1 - e^{-bx}) \qquad (1)$$

where, $f(0)$ is the yield in the absence of fertilization and a and b are constants, determined with the least squares method by comparison with the experimental data.

Starting from the model proposed by Mitscherlich, the present study proposes a new model, with greater accuracy in evaluating the relation between fertilizers and yield. The model was used for assessing the benefit for the purpose of optimizing the fertilization of wheat crops. Coefficient determination and graphic representation were made with MuPAD Pro 4.0 software and ANOVA test was employed for the statistical analysis of data.

RESULTS

In the case when one-nutriment fertilizers are applied, the relation among fertilizer and yield is unifactorial, and the mathematical relation between dose x and agricultural yield *f(x)* being best given by the Mitscherlich function (1). The experimental results are presented in Table 1.

The contribution of the two factors (N and PK) to the yield is significant. The results of the ANOVA test show that the yield differences between the experimental treatments are significant, with statistical assurance for Alpha = 0.001 (Table 2).

For the situations where only one nutrient is provided through fertilization, we propose a model for assessing the dependence of the yield on the dose of fertilizer. This model is designed with the help of relation

$$f(x) = f(0) + a \cdot \tanh(bx) \quad (1)$$

If we consider benefit as the difference between yield value V and costs C, we will obtain the maximum for the value that annuls its derivative.

For the case when two fertilizers are used, we admit the generalization:

$$f(x,y) = f(0,0) + \frac{a_1 \tanh(b_1 x) + a_2 \tanh(b_2 y) + a_3 \tanh(b_1 x)\tanh(b_2 y)}{1 + \tanh(b_1 x)\tanh(b_2 y)} \qquad (2)$$

where, $f(0,0)$ is the yield for the control crop, terms two and

Table 1. Wheat yield (kg ha^{-1}) under the influence of mineral fertilizers (N and PK) *Alex* cultivar, Timişoara, 2008-2010.

PK \ N	N0	N50	N100	N150	N200
	(kg ha^{-1})				
P0K0	2348±103.69	2865±112.51	3357±107.41	3673±115.52	3822±141.73
P50K50	2977±106.86	3865±138.11	4585±149.78	5110±112.69	5460±121.66
P100K100	3214±125.80	4186±97.74	4857±115.70	5610±143.56	6052±126.30
P150K150	3358±121.32	4428±118.90	5565±141.80	5978±122.52	6110±144.42

Table 2. ANOVA: Two-Factor without Replication

Source of Variation	SS	Df	MS	F	P-value	F crit
Rows (PK)	10063298	3	3354433	79.5353	8.48E-07	13.9018
Columns (N)	5391682	3	1797227	42.6132	1.21E-05	13.9018
Error	379579	9	42175			
Total	15834558	15				

Alpha = 0.001

three represent the separate contribution of nitrogen fertilizer and phosphorus:potassium fertilizer respectively to the total yield.

The last term represents the involvement of the interaction between nitrogen and phosphorus:potassium complex to the total yield. As working assumption in the case of the generalization to two variables, in order to calculate all three types of fertilizers (nitrogen, phosphorus and potassium fertilizers), but without modeling with three variables, we take the phosphorus:potassium dose as one variable which considers the two elements in equal proportions for any dose used in the experiment. There are two observations to be made: the first is that interaction appears as zero if one of the fertilizers is missing, which is natural. The other is that, if we work with constant values for one type of fertilizer, then the model is unifactorial. Constants a_1, a_2, a_3, b_1 and b_2 are determined with the least squares method, by comparison with the experimental data, using MuPAD Pro 4.0 software.

After applying this method, we obtain the following values for the constants: a_1=1714.28, a_2=1022.22, a_3=4864.15 b_1=0.006644, b_2=0.006894.

Figures 2 to 5 represent the experimental points found in Table 1 and the theoretic curves given by relation (2) for nitrogen variation. Figures 6 to 10 show the same theoretic curves given by relation (2) for the variation of the phosphorus:potassium complex.

Figures 2 to 10 confirm good concordance between the experimental data and the theoretical curves. The experimental fertilizer doses show on the abscissae axis, while the yield values appear on the ordinate axis. Simultaneously, we well made a three-dimensional representation of the theoretical yield in relation to the two factors considered in the model, together with the experimental data (Fig. 11). The three-dimensional representation shows good concordance involving the experimental data and the theoretical surface, as well.

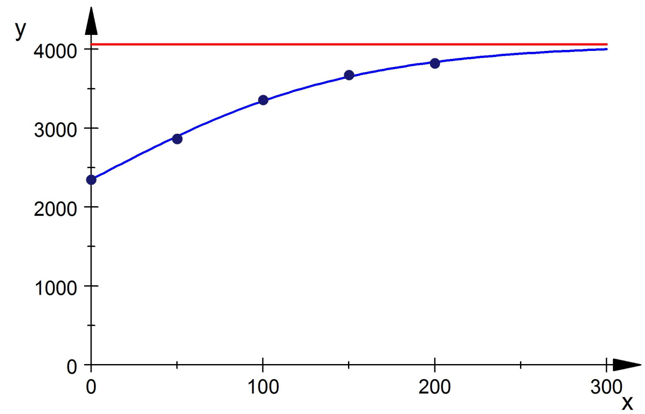

Figure 2. Dependence of wheat yield on the nitrogen dose for P_0K_0 (*x* - nitrogen doses kg ha^{-1} active substance, y - yield kg ha^{-1})

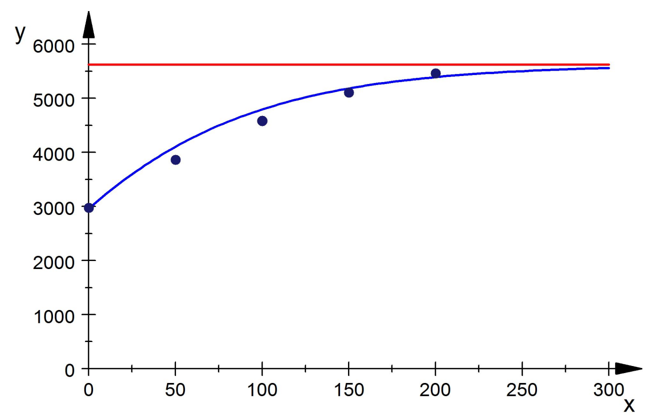

Figure 3. Dependence of wheat yield on the nitrogen dose for $P_{50}K_{50}$ (x - nitrogen doses kg ha^{-1} active substance, y - yield kg ha^{-1})

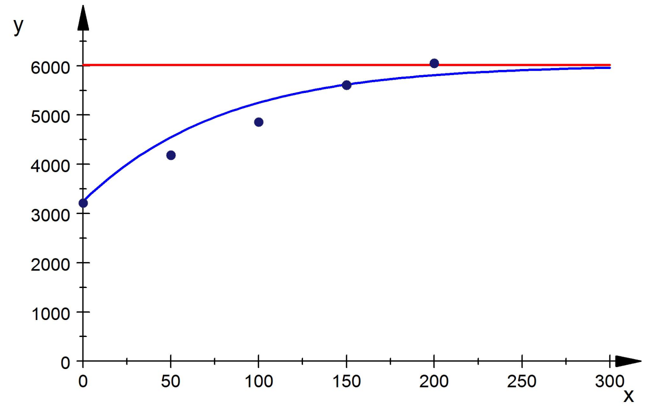

Figure 4. Dependence of wheat yield on the nitrogen dose for $P_{100}K_{100}$ (x - nitrogen doses kg ha^{-1} active substance, y - yield kg ha^{-1})

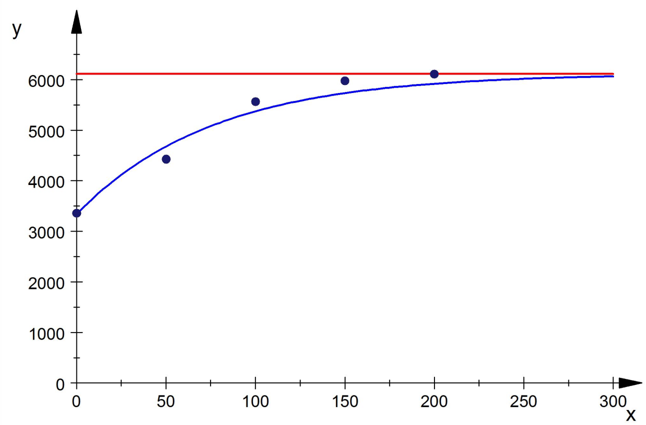

Figure 5. Dependence of wheat yield on the nitrogen dose for $P_{150}K_{150}$ (x - nitrogen doses kg ha^{-1} active substance, y - yield kg ha^{-1})

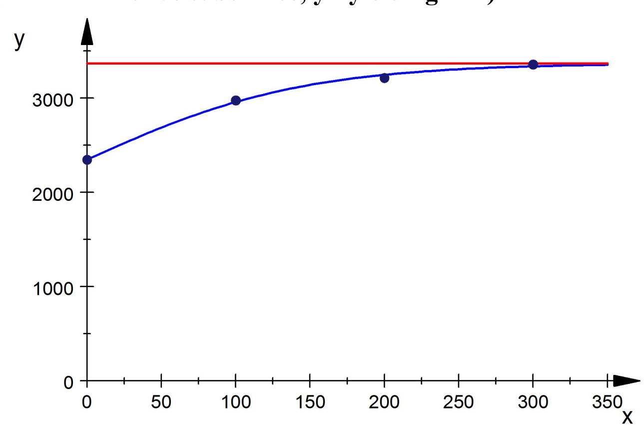

Figure 6. Dependence of wheat yield on the phosphorus:potassium dose for N_0 (x - phosphorus:potassium doses kg ha^{-1} active substance, y - yield kg ha^{-1})

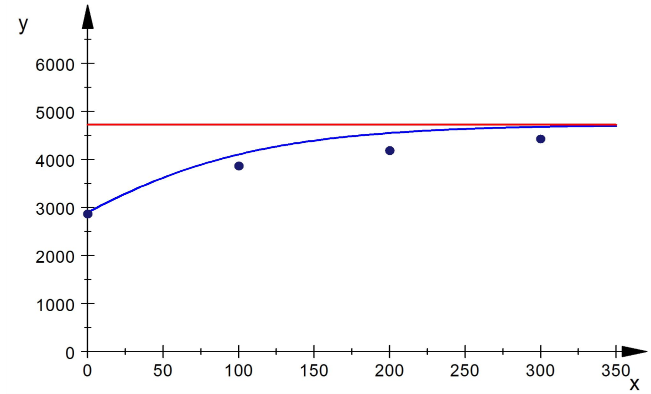

Figure 7. Dependence of wheat yield on the phosphorus:potassium dose for N_{50} (x - phosphorus:potassium doses kg ha^{-1} active substance, y - yield kg ha^{-1})

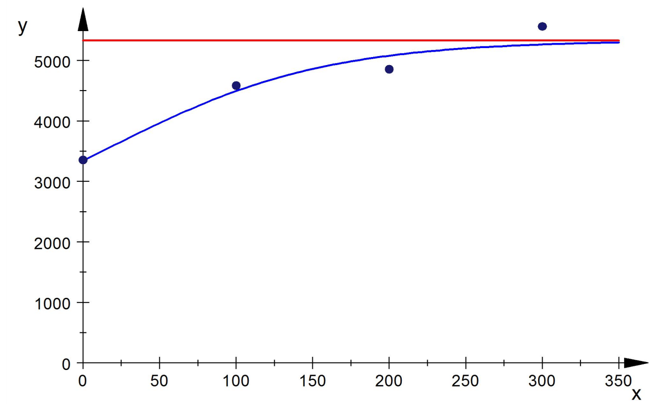

Figure 8. Dependence of wheat yield on the phosphorus:potassium dose for N_{100} (x - phosphorus:potassium doses kg ha^{-1} active substance, y - yield kg ha^{-1})

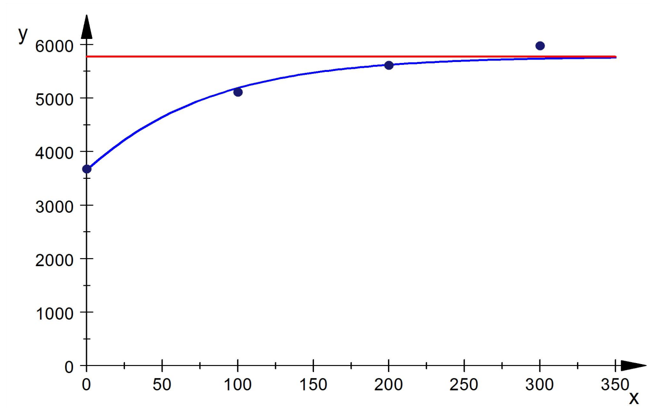

Figure 9. Dependence of wheat yield on the phosphorus:potassium dose for N_{150} (x - phosphorus:potassium doses kg ha^{-1} active substance, y - yield kg ha^{-1})

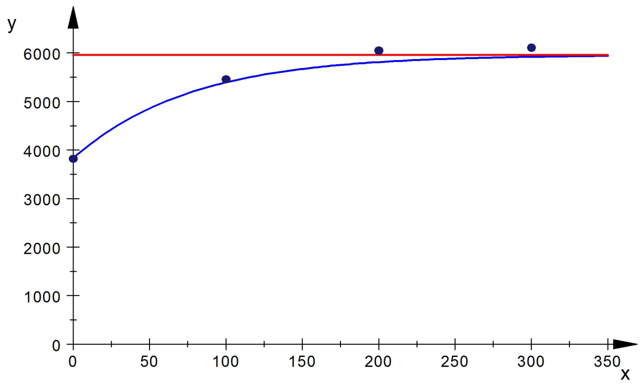

Figure 10. Dependence of wheat yield on the phosphorus:potassium dose for N_{200} (x - phosphorus:potassium doses kg ha^{-1} active substance, y - yield kg ha^{-1})

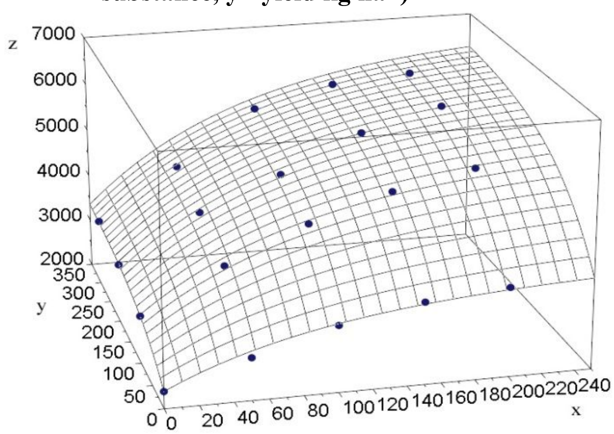

Figure 11. Three-dimensional representation of function (2) and of experimental data, (x - nitrogen doses kg ha^{-1} active substance, y - phosphorus: potassium doses kg ha^{-1} active substance, z - yield kg ha^{-1})

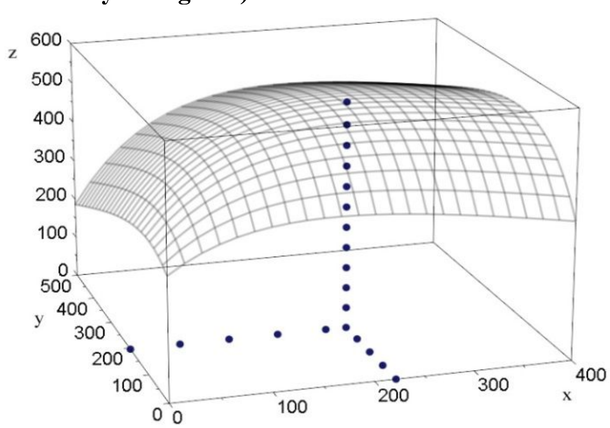

Figure 12. Graphical method for determining the optimal solutions, (x - nitrogen doses kg ha^{-1} active substance, y - phosphorus:potassium doses kg ha^{-1} active substance, z-yield kg ha^{-1})

From an economic point of view, in this bidimensional case, benefit B is written:

$$B = qf(x, y) - p_1 x - p_2 y \qquad (3)$$

where: p_1 is the price of nitrogen (in Euro), p_2 is the price of the phosphorus - potassium complex (in Euro), and q is the market price of a kilo of wheat (in Euro, at the London stock exchange).

It is known that the benefit is maximum if the partial derivatives are null, meaning:

$$\begin{cases} \dfrac{a_1 + a_3 \tanh(b_2 y) - a_2 \tanh^2(b_2 y)}{\cosh^2(b_1 x)(1 + \tanh(b_1 x)\tanh(b_2 y))^2} - \dfrac{p_1}{q b_1} = 0 \\ \dfrac{a_1 + a_3 \tanh(b_1 x) - a_1 \tanh^2(b_1 x)}{\cosh^2(b_2 y)(1 + \tanh(b_1 x)\tanh(b_2 y))^2} - \dfrac{p_2}{q b_2} = 0 \end{cases} \qquad (4)$$

For the price values q = 0.18 €/kg, p_1=1.05€/kg, and p_2=1.12 €/kg the solutions for system (4) are x =146.15 kg active substance N ha^{-1} and y = 117.80 kg active substance PK ha^{-1}. The fixed costs related to the application of fertilizers and to harvesting have not been taken into consideration, because the optimal solution x, y is not affected by these variations. The derivatives of the constants are zero and so their influence is annulled. When we represent function (3) graphically, we can see that the coordinates for the maximum are precisely the optimal solutions of the system (4) (Fig. 12).

DISCUSSION

Specialized literature includes various authors who started their research from Mitscherlich's unifactorial model, (1909, 1913). Among them, Harmsen (2000a, 2000b), Harmsen *et al.* (2001), Nijlandl *et al.* (2008) applied the function in the initial variant for a single factor:

$$f(x) = f(0) + a(1 - e^{-bx})$$

Still, relatively soon after the first model, researchers felt the need to continue studies in this field by expanding the model to more than one factor. Thus, Baule (1917) and Mitscherlich (1956) gave generalizations of this model, but these were made empirically to factors by a direct product of the type:

$$\frac{Y}{Y_x} = (1 - e^{-bN_1})(1 - e^{-bN_2})...(1 - e^{-bN_n})$$

Other contributions for the expansion of the model can be cited, such as Harmsen (2000a), where the author proposes the application of Taylor series for the unifactorial model, thus transforming them from continuous models to discrete models, which are much easier to use in agricultural practice. However, their modeling is less accurate and their prediction is less certain.

The unifactorial model we started from in the present paper is also of the Mitscherlich type. Expanding to two variables was also made empirically, but to a function of the type:

$$f(x,y) = f(0,0) + \frac{a_1 \tanh(b_1 x) + a_2 \tanh(b_2 y) + a_3 \tanh(b_1 x)\tanh(b_2 y)}{1 + \tanh(b_1 x)\tanh(b_2 y)}$$

The advantage of the model proposed in this paper, compared to the model proposed by Harmsen, is that it takes into consideration the independent action of the two factors *x* and *y*, and also the results of their interaction on the wheat yield. The function *f (x,y)* represents the yield in relation to the two types of fertilizers.

The main, obligatory characteristic of the function we propose is that, if one of the variables equals zero (one fertilizer is missing from the initial assumption), then the function in relation (2) is reduced to a hyperbolic unifactorial function of the type (1). This does not show in any of the models proposed by other authors. Although given empirically, this function is supported by the experimental data in Table 1, which were as well represented in figures 2 to 11.

Conclusions**:** One of the best theoretical links between the fertilizer dose (*x*) and yield is given by the Mitscherlich function, relation (1). For its generalization to two variables, for two types of fertilizers respectively, was admitted the function given by relation (3). Defining benefit (*B*) as the difference between the yield value (*V*) and cost (*C*), the maximum of this economic indicator is obtained by solving the resulting equation by cancelling the partial derivatives. In this way the optimal solutions for fertilizer doses were obtained. The model obtained allows us to predict the yield for a certain fertilizer dose which is possible to be provided by the crop technology, or it allows us to estimate/calculate a certain dose for obtaining a certain yield, depending on how intensive the agricultural system is. In regard to the universality of this model, except for the determination of the constants that depend on experiment specificity, it can be appreciated that this type of function is useful both for yield prediction and for the determination of certain economic indicators.

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